CS 188: Artificial Intelligence
Fall 2009

Lecture 3: A* Search
9/3/2009

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Many slides from Dan Klein

Announcements

- Assignments:
  - Project 0 (Python tutorial): due Thursday 1/28
  - Written 1 (Search): due Thursday 1/28
  - Project 1 (Search): to be released today, due Thursday 2/4
    - You don't need to submit answers to P1 discussion questions
  - 5 slip days for projects; up to two per deadline
  - Try pair programming, not divide-and-conquer
- Study materials
  - Slides, Section materials, Assignments
  - Book

Office hours, Section

- Drop-in lab times: Wed 1/26 4-5pm in 271 Soda
- Office hours posted on the course website
- Sections starting this week:
  - Working though exercises are key for your understanding
  - Section handout contains several exercises similar to written 1
  - Solutions will be posted Wed 1pm (after last section)
  - Section 101: Tue 3-4pm
  - Section 104: Tue 4-5pm
  - Section 102: Wed 11-noon
  - Section 103: Wed noon-1pm

Today

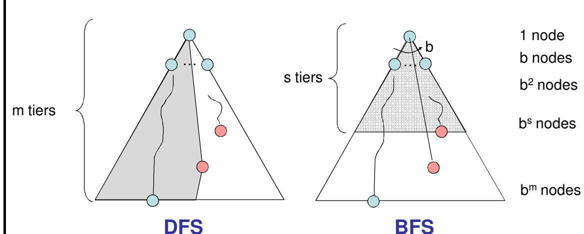
- Iterative deepening
- Uniform cost search
- A* Search
- Heuristic Design

Recap: Search

- Search problem:
  - States (configurations of the world)
  - Successor function: a function from states to lists of (state, action, cost) triples; drawn as a graph
  - Start state and goal test
- Search tree:
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)
- Search Algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)

DFS and BFS

Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	Y	N	$O(b^m)$	$O(bm)$
BFS		Y	N*	$O(b^{s+1})$	$O(b^{s+1})$

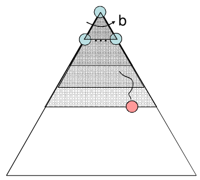


Iterative Deepening

Iterative deepening uses DFS as a subroutine:

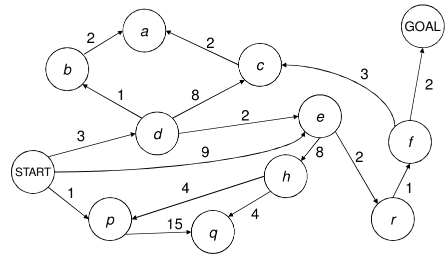
1. Do a DFS which only searches for paths of length 1 or less.
2. If "1" failed, do a DFS which only searches paths of length 2 or less.
3. If "2" failed, do a DFS which only searches paths of length 3 or less.

….and so on.



Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	Y	N	$O(b^m)$	$O(bm)$
BFS		Y	N*	$O(b^{s+1})$	$O(b^{s+1})$
ID		Y	N*	$O(b^s)$	$O(bs)$

Costs on Actions



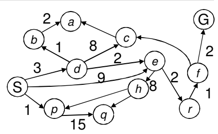
Notice that BFS finds the shortest path in terms of number of transitions. It does not find the least-cost path.

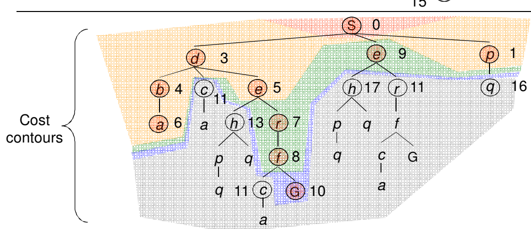
We will quickly cover an algorithm which does find the least-cost path.

Uniform Cost Search

Expand cheapest node first:

Fringe is a priority queue





Priority Queue Refresher

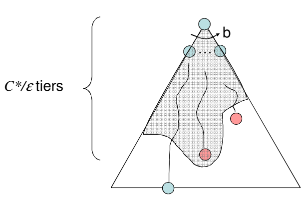
- A priority queue is a data structure in which you can insert and retrieve (key, value) pairs with the following operations:

pq.push(key, value)	inserts *(key, value)* into the queue.
pq.pop()	returns the key with the lowest value, and removes it from the queue.

- You can decrease a key's priority by pushing it again
- Unlike a regular queue, insertions aren't constant time, usually O(log *n*)
- We need priority queues for cost-sensitive search methods

Uniform Cost Search

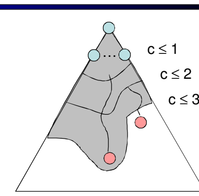
Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	Y	N	$O(b^m)$	$O(bm)$
BFS		Y	N	$O(b^{s+1})$	$O(b^{s+1})$
UCS		Y*	Y	$O(b^{C^*/\varepsilon+1})$	$O(b^{C^*/\varepsilon+1})$

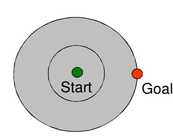


** UCS can fail if actions can get arbitrarily cheap*

Uniform Cost Issues

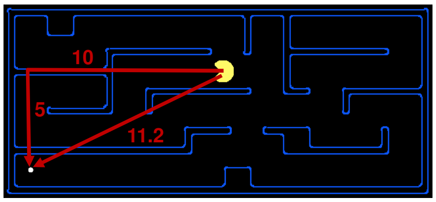
- Remember: explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about goal location



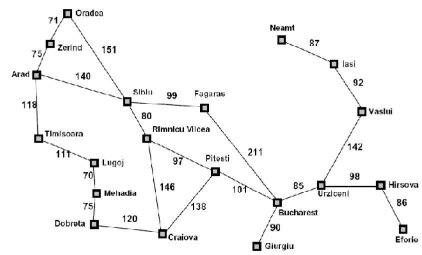


Search Heuristics

- Any *estimate* of how close a state is to a goal
- Designed for a particular search problem
- Examples: Manhattan distance, Euclidean distance



Heuristics

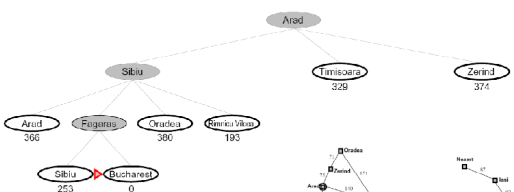


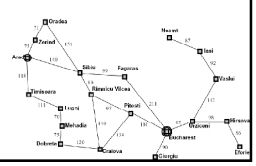
Straight line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Best First / Greedy Search

- Expand the node that seems closest…

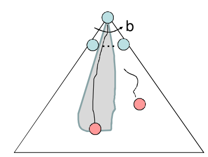


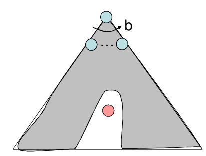


- What can go wrong?

Best First / Greedy Search

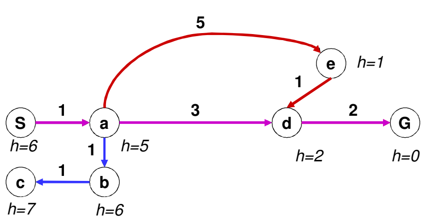
- A common case:
  - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS in the worst case
  - Can explore everything
  - Can get stuck in loops if no cycle checking
- Like DFS in completeness (finite states w/ cycle checking)





Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Best-first orders by goal proximity, or *forward cost* h(n)

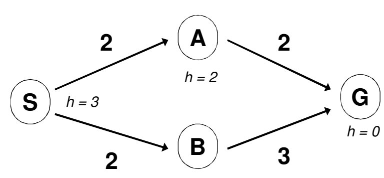


- A* Search orders by the sum: f(n) = g(n) + h(n)

Example: Teg Grenager

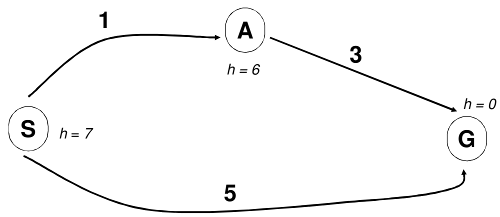
When should A* terminate?

- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

Is A* Optimal?



- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

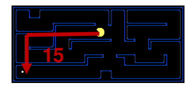
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Example:

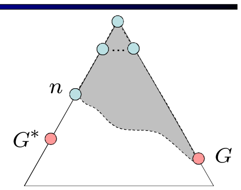


- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A*: Blocking

Notation:

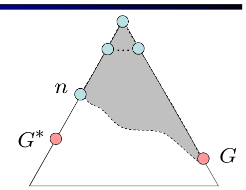
- g(n) = cost to node n
- h(n) = estimated cost from n to the nearest goal (heuristic)
- f(n) = g(n) + h(n) = estimated total cost via n
- G*: a lowest cost goal node
- G: another goal node



Optimality of A*: Blocking

Proof:

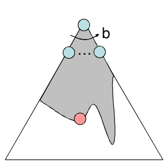
- What could go wrong?
- We'd have to have to pop a suboptimal goal G off the fringe before G*
- This can't happen:
  - Imagine a suboptimal goal G is on the queue
  - Some node *n* which is a subpath of G* must also be on the fringe (why?)
  - *n* will be popped before G



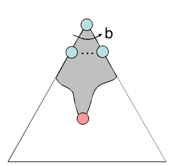
$$f(n) = g(n) + h(n)$$
$$g(n) + h(n) \leq g(G^*)$$
$$g(G^*) < g(G)$$
$$g(G) = f(G)$$
$$f(n) < f(G)$$

Properties of A*

Uniform-Cost

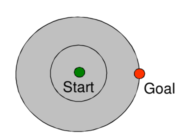


A*

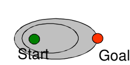


UCS vs A* Contours

- Uniform-cost expanded in all directions



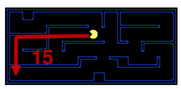
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



[demo: countours UCS / A*]

Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available



- Inadmissible heuristics are often useful too (why?)

Trivial Heuristics, Dominance

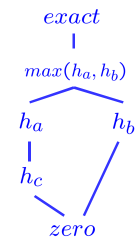
- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic

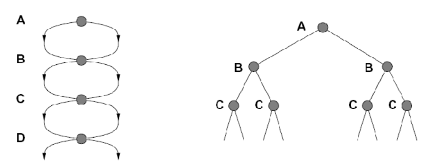


Other A* Applications

- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

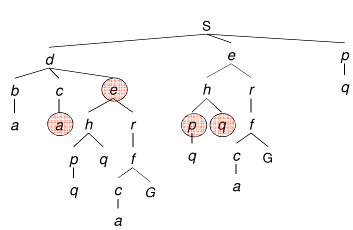
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work. Why?



Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



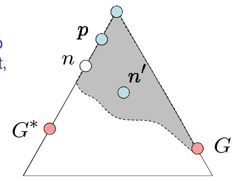
Graph Search

- Idea: never expand a state twice
- How to implement:
  - Tree search + list of expanded states (closed list)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state is new
- Python trick: store the closed list as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

Optimality of A* Graph Search

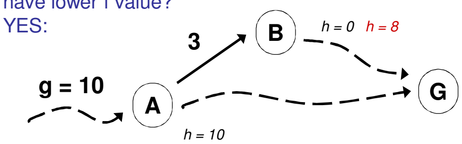
Proof:

- New possible problem: nodes on path to G* that would have been in queue aren't, because some worse *n'* for the same state as some *n* was dequeued and expanded first (disaster!)
- Take the highest such *n* in tree
- Let *p* be the ancestor which was on the queue when *n'* was expanded
- Assume *f(p) < f(n)*
- *f(n) < f(n')* because *n'* is suboptimal
- *p* would have been expanded before *n'*
- So *n* would have been expanded before *n'*, too
- Contradiction!



Consistency

- Wait, how do we know parents have better f-values than their successors?
- Couldn't we pop some node *n*, and find its child *n'* to have lower f value?
- YES:



- What can we require to prevent these inversions?
- Consistency: $c(n, a, n') \geq h(n) - h(n')$
- Real cost must always exceed reduction in heuristic

Optimality

- Tree search:
  - A* optimal if heuristic is admissible (and non-negative)
  - UCS is a special case (h = 0)
- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)
- Consistency implies admissibility
- In general, natural admissible heuristics tend to be consistent

Summary: A*

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible heuristics
- Heuristic design is key: often use relaxed problems